# ON THE $C_{p}$ TO $C_{v}$ CONVERSION OF SOLID LINEAR MACROMOLECULES II 

R. Pan*, Manika Varma Nair and B. Wunderlich

CHEMISTRY DIVISION OF OAK RIDGE NATIONAL LABORATORY AND DEPARTMENT OF CHEMISTRY, UNIVERSITY OF TENNESSEE KNOXVILLE, TENNESSEE 37996-1600<br>*DEPARTMENT OF CHEMISTRY, RENSSELAER POLYTECHNIC INSTITUTE TROY, NEW YORK 12180, U.S.A.

(Received September 28, 1988; in revised form November 28, 1988)


#### Abstract

A modification is proposed for the Nernst-Lindemann equation that is used to convert calculated heat capacities at constant pressure $\left(C_{p}\right)$ to heat capacities at constant volume $\left(C_{v}\right)$ for solid, linear macromolecules. the constant $A_{0}$ per mole of repeating unit in this equation is derived by taking into account the variable number of vibrators excited at different temperatures. With the new equation it is possible to calculate $C_{p}$ for solid polymers over a wider temperature range. The constant is calculated for solid polymers from experimental thermal expansivity, isothermal compressibility and heat capacity data obtained from the literature. An average value of $(3.9 \pm 2.4) \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / \mathbf{J}$ was obtained for $A_{0}$ (new) from data on 22 solid polymers. This average value may be used as a universal constant in case no experimental data on compressibility and expansivity are available for computation of $\boldsymbol{A}_{0}$. The remaining variation of $A_{0}$ (new) with temperature is discussed and example calculations are shown for polyethylene. Effects of premelting and possibly large-amplitude motion are discovered for polyethylene in the temperature range 290 to 410 K .


## Introduction and derivation of the new equation

The use of the Nernst-Lindemann equation to convert heat capacity at constant pressure $\left(C_{p}\right)$ to heat capacity at constant volume $\left(C_{v}\right)$ for solid, linear macromolecules was discussed earlier [1].

$$
\begin{equation*}
C_{p}-C_{v}=A_{0} C_{p}^{2} T / T_{m}^{0} \tag{1}
\end{equation*}
$$

It was suggested that in the usual ab'sence of $p V T$ data over a wide temperature range, it is permissible to convert $C_{v}$, calculated from an approximate vibrational spectrum, to $C_{p}$ by using a universal constant $A_{0}$. An average value of $5.11 \times 10^{-3}(\mathrm{k} \mathrm{mol}) / \mathbf{J}$ (computed per mole of heavy atoms) was obtained for $A_{0}$ from $p V T$ data of several polymeric crystals and glasses at a temperature of about

298 K . This value of $A_{0}$ is numerically close to the original Nernst-Lindemann constant derived for metals and salts [2].

At ATHAS, our laboratory for $A$ dvanced $T H$ ermal $A$ nalysiS, the linking of heat capacities of nearly 100 macromolecules to their vibrational spectra has recently been completed [3]. The heat capacity $C_{p}$ obtained from calculated $C_{v}$ was found to agree to better than $\pm 3 \%$ with the experimental heat capacity $C_{p}$ up to the glass or melting temperatures. Calculations carried out beyond these temperatures showed a rather sharp, unrealistic upturns in the calculated $C_{p}$. Since $C_{v}$ showed only a moderate increase with temperature, the change in the calculated $C_{p}$ was assumed to be in error and attributed to the limit of usefulness of the Nernst-Lindemann equation. Naturally such error at high temperature casts also doubt on the applicability of the equation at lower temperature.

It was, therefore, considered worthwhile to modify the assumptions made earlier in deriving $A_{0}$. The Nernst-Lindemann equation was derived originally for relatively heavy, monatomic solids [2] with $T_{m}^{0}$ representing the equilibrium melting temperature. In finding a suitable constant $A_{0}$ for solid, linear macromolecules it was assumed that only the vibrations associated with the heavy atoms (such as $\mathbf{C}, \mathrm{N}$ and $O$ ) contribute to the heat capacity:

$$
\begin{equation*}
A_{0}=A_{0}(\mathrm{old}) /\left({ }^{\#} \text { heavy atoms }\right) \tag{1a}
\end{equation*}
$$

This can only be a first approximation since at higher temperatures vibrations, such as $\mathrm{C}-\mathrm{H}, \mathrm{N}-\mathrm{H}$ or $\mathrm{O}-\mathrm{H}$ stretching and bending, also contribute to the heat capacity. In fact, there is a simple measure of the number of vibrators excited at any given temperature, given by the heat capacity at constant volume itself:

$$
\begin{equation*}
C_{v}=3 n R, \tag{2}
\end{equation*}
$$

where $n$ is the average number of excited vibrators.
Based on Eq. (2) a better, temperature dependent value can be derived for $A_{0}$ :

$$
\begin{equation*}
A_{0}=A_{0}(\text { new }) / n, \tag{3}
\end{equation*}
$$

where $A_{0}$ is expressed per mole of repeating unit and $A_{0}$ (new) represents the value for one mole of vibrationally excited atoms [1]. Substituting equations (2) and (3) into (1) one gets:

$$
\begin{equation*}
C_{p}-C_{v}=3 R A_{0}(\text { new }) C_{p}^{2} T /\left(C_{v} T_{m}^{0}\right) \tag{4a}
\end{equation*}
$$

and since in the original justification of Eq. (1) $C_{p}$ was assumed to be equal to $C_{v}$ one can also write

$$
\begin{equation*}
C_{p}-C_{v}=3 R A_{0}(\text { new }) C_{p} T / T_{m}^{0} \tag{4b}
\end{equation*}
$$

or

$$
\begin{equation*}
C_{p}-C_{v}=3 R A_{0}(\text { new }) C_{v} T / T_{m}^{0} \tag{4c}
\end{equation*}
$$

the differences between the three expressions are small at low temperature, typically $0.06 \%$ and $0.12 \%$ in $A_{0}$ for polyethylene at 298.15 K , much less than the changes in $A_{0}$ from sample to sample or with temperature. For computational reasons all further discussions will make use of Eq. (4a).

In this paper the results of $C_{v}$ to $C_{p}$ conversions of solid macromolecules are presented using this new equation. A value of the constant $A_{0}($ new $)$ is proposed on the basis of experimental thermal expansivity ( $\alpha$ ) and isothermal compressibility ( $\beta$ ) obtained from the literature for various polymers over wider ranges of temperature. The connection between $A_{0}$ (new) and $\alpha$ and $\beta$ is given by comparison of the thermodynamic relationship
with Eq. (4a).

$$
C_{p}-C_{v}=T V \alpha^{2} / \beta
$$

## Data on $p V T$ for macromolecules

Although many studies have been reported on $p V T$ properties of various macromolecules, the dilatometry is often limited to semicrystalline and glassy states. Equations of state have been used to obtain crystal data from experimental information on semicrystalline materials [4,5]. The combination of melt and crystal theories have shown good agreement between theoretically predicted and experimentally obtained specific volumes. A two-phase model of additivity of crystal and amorphous volumes was also used by Tsujita et al. [6] to obtain $p V T$ data of crystalline regions from the information on semicrystalline solid and melt data on polyoxyethylene and polyoxytetramethylene. According to this model:

$$
\begin{align*}
V & =(1-x) V_{a}+x V_{c}  \tag{5}\\
\alpha & =(1-x) \alpha_{a} V_{a} / V+x \alpha_{c} V_{c} / V  \tag{6}\\
\beta & =(1-x) \beta_{a} V a / V+x \beta_{c} V_{c} / V \tag{7}
\end{align*}
$$

The degree of crystallinity, $x$, was assumed to be invariant with temperature and pressure, $V$ is the molar volume of amorphous ( $a$ ), crystalline ( $c$ ) or the semicrystalline (no subscript) samples. The thermal expansivity ( $\alpha$ ) and isothermal compressibility $(\beta)$ are defined similarly. For the amorphous regions in semicrystalline samples $\alpha_{a}$ and $\beta_{a}$ were extrapolated from the melt. The glass transition temperature, $T_{g}$, was the lower limit of this extrapolation. Using these methods all literature data were reevaluated using either presented equations or newly derived best fits. The data were extracted using references [1, 6-20].

## Results and discussion

For various polymers in the glassy, crystalline and semicrystalline states the resulting $A_{0}$ (new) values are shown in Tables 1 and 2 and compared to values of $A_{0}$ (old). Note that in all tables more significant figures than warranted by the precision of the experiments are listed. This is done for computational reasons so that the original data can be reproduced and proper error assessment made after further use of the data. Using the new Eq. (4a) an average value of $(3.9 \pm 2.4) \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / \mathrm{J}$ is obtained. The old method using Eq. (1a) gave with the present data collection a value of $(5.4 \pm 3.3) \times 10^{-3}(\mathrm{k} \mathrm{mol}) / \mathrm{J}$. to obtain these averages, a weight of two was given to all $A_{0}$ values corresponding to a wider temperature range, while $A_{0}$ from a single temperature was given a weight of one. The average of $A_{0}$ (new) glassy polymers is lower than that for crystalline and semicrystalline polymers. The variation from polymer to polymer does not seem to

Table 1 Calculated $A_{0}$ for various glassy polymers

| Polymers* | Temp. range of experimental $p V T$ data, K | $\begin{aligned} & \text { Melting } \\ & \text { temp.,** } \\ & \mathrm{K} \end{aligned}$ | Average $A_{0} \times 10^{3},$ <br> $\mathrm{K} \mathrm{mol}^{-1}$ <br> (New) | $\begin{gathered} A_{0} \times 10^{3} \\ \mathrm{~K} \mathrm{~mol} \mathrm{~J} \\ (\text { Old }) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| Poly(vinyl acetate) (6) | 240-290 | 446 | $3.25 \pm 0.66$ | $5.18 \pm 1.16$ |
| Poly(methyl methacrylate) (7) | 340-370 | 450 | $1.78 \pm 0.21$ | $2.29 \pm 0.34$ |
| Poly( $4,4^{\prime}$-isopropylidene diphenyl carbonate) (19) | 310-420 | 608.2 | $3.21 \pm 0.57$ | $4.93 \pm 1.60$ |
| Polystyrene (8) | 280-340 | 516.2 | $2.46 \pm 0.33$ | $3.88 \pm 0.27$ |
| Poly(2,6-dimethyl-1,4-phenylene oxide) (9) | 303 | 535 | 2.34 | 3.61 |
| Poly( $n$-butyl methacrylate) (10) | 290-370 | 439 | $2.63 \pm 0.53$ | $3.26 \pm 0.49$ |
| Poly(vinyl chloride) (3) | 220-320 | 546 | $2.64 \pm 0.59$ | $4.0 \pm 1.67$ |
| Poly(vinyl fluoride) (3) | 290-310 | 503.2 | $10.18 \pm 0.51$ | $15.67 \pm 1.38$ |
| Poly(chloro trifluoro--ethylene) (6) | 290-320 | 493 | $4.30 \pm 0.23$ | $6.88 \pm 0.58$ |
| Poly(vinyl alcohol) (3) | 280-300 | 521 | $3.73 \pm 0.37$ | $4.54 \pm 0.68$ |
| Poly(ethylene terephthalate) (14) | 300-330 | 553 | $0.81 \pm 0.03$ | $1.21 \pm 0.10$ |
| ```Poly(oxy-1,4-phenylene- -sulfonyl-1,4-phenylene oxy-1,4-phenylene(1-methylidene)- 1,4-phenylene) (32)``` | 300-450 | 687 | $3.15 \pm 0.53$ | $4.62 \pm 1.19$ |
| Average $A_{0}$ (new) $=3.42 \pm 2.3 \mathrm{I}$ <br> Average $A_{0}(\mathrm{old})=5.07 \pm 3.64$ |  |  |  |  |

[^0]Table 2 Calculated $A_{0}$ for various crystalline and semicrystalline polymers

| Polymers* | Temp. range <br> of experimental <br> $p V t$ data, K | Melting <br> temp.,** <br> K | Average <br> $A_{0} \times 10^{3}$ <br> $\mathrm{~K} \mathrm{~mol} \mathrm{~J}{ }^{-1}$ <br> (new) | $A_{0} \times 10^{3}$ <br> K mol J <br> (old) |
| :--- | :---: | :---: | :---: | :---: |
| Polyethylene (1) (c)*** | $290-310$ | 414.6 | $3.18 \pm 0.98$ | $3.89 \pm 1.20$ |
| Polypropylene (3) (c) | $300-370$ | 460.7 | $5.93 \pm 0.51$ | $6.59 \pm \pm 1.12$ |
| Polyoxymethylene (2) (s) | 293 | 457 | 2.54 | 3.52 |
| Polyoxyethylene (3) (c) | $230-330$ | 342 | $0.74 \pm 0.36$ | $1.04 \pm 0.46$ |
| Polyoxytetramethylene (5) (s) | $240-270$ | 330 | $3.92 \pm 0.28$ | $3.94 \pm 0.27$ |
| Poly(1-butene) (4) (s) | $300-320$ | 411.2 | $8.80 \pm 1.15$ | $10.20 \pm 2.11$ |
| Poly(4-methyl-1-pentene) (6) (s) | $290-300$ | 523 | $5.85 \pm 0.08$ | $6.66 \pm 0.13$ |
| Polytetrafluoroethylene (3) (s) | $300-390$ | 605 | $4.578 \pm 0.63$ | $7.557 \pm 1.49$ |
|  | $380-500$ | 605 | 3.796 | 4.86 |
| Poly(vinylidene fluoride) (4) (s) | 293 | 483.2 | 2.55 | 3.76 |
| Nylon 6 (8) (s) | 293 | 533 | 8.01 | 11.02 |

Average $A_{0}$ (new) $=4.56 \pm 2.33$
Average $A_{0}(o l d)=5.67 \pm 2.84$

[^1]Table 3 Calculated $A_{0}$ for glassy Poly(4,4-isopropylidene diphenylene carbonate)

| Temp., <br> K | $V_{a} \times 10^{5}$, <br> $\mathrm{m}^{3} / \mathrm{mol}$ | $\alpha_{a} \times 10^{4}$, <br> $\mathrm{K}^{-1}$ | $\beta_{a} \times 10^{10}$, <br> $\mathbf{P a}^{-1}$, | $C_{p}$, <br> $\mathbf{J ~ K}^{+1} \mathrm{~mol}^{-1}$ | $\mathrm{J} \mathrm{K}^{-1}$, <br> $\mathrm{mol}^{-1}$ | $A_{0} \times 10^{3}$ <br> $(\mathrm{new})$, <br> $\mathrm{K} \mathrm{mol} \mathrm{J}^{-1}$ | $A_{0} \times 10^{3}$ <br> $(\mathrm{old})$, <br> K mol J |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 310.00 | 21.310 | 2.624 | 2.538 | 316.900 | 298.976 | 4.197 | 7.704 |
| 320.00 | 21.366 | 2.617 | 2.605 | 327.100 | 309.121 | 3.958 | 7.026 |
| 330.00 | 21.422 | 2.611 | 2.674 | 337.400 | 319.384 | 3.735 | 6.417 |
| 340.00 | 21.478 | 2.604 | 2.745 | 347.700 | 329.663 | 3.527 | 5.871 |
| 350.00 | 21.533 | 2.597 | 2.817 | 358.100 | 340.058 | 3.333 | 5.379 |
| 360.00 | 21.589 | 2.590 | 2.892 | 368.500 | 350.467 | 3.152 | 4.936 |
| 370.00 | 21.645 | 2.584 | 2.968 | 379.100 | 361.090 | 2.982 | 4.532 |
| 380.00 | 21.701 | 2.577 | 3.047 | 389.700 | 371.726 | 2.823 | 4.167 |
| 390.00 | 21.757 | 2.570 | 3.127 | 400.300 | 382.374 | 2.674 | 3.838 |
| 400.00 | 21.813 | 2.564 | 3.210 | 411.100 | 393.234 | 2.534 | 3.536 |
| 410.00 | 21.869 | 2.557 | 3.295 | 421.900 | 404.105 | 2.403 | 3.263 |
| 420.00 | 21.925 | 2.551 | 3.382 | 480.300 | 462.585 | 2.062 | 2.446 |

be much different from the previous calculation [1], i.e. $A_{0}$ (new) is also an approximate, but not a precise, universal constant. For polymers where data over a wider temperature range are available, it is possible to check into the variation of $A_{0}$ with temperature. For an exact fit of Eq. (4a), $A_{0}$ should be a constant. In case of glassy polymers, the improved, but still approximate nature of this equation was evident from a remaining, smaller change of $A_{0}$ (new) than of $A_{0}$ (old) with temperature. As an example the results of the calculations for poly(4,4isopropylidene diphenylene carbonate) are depicted in Table 3 [14]. A decrease in $A_{0}$ with an increase in temperature was also observed for the other glassy polymers with exceptions of polystyrene [17], poly(methyl methacrylate) [7, 15] and poly(nbutyl methacrylate) [7] in which $A_{0}$ showed a slight increase with temperature. In addition to these changes, the conventional atactic PMMA showed a jump in $A_{0}$ between $330-340 \mathrm{~K}$ [the value of $A_{0}$ (new) changed from $1.85 \times 10^{-3}$ to $\left.2.369 \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / \mathrm{J}\right]$. A distinct break, as reported earlier, was also observed in molar volume and expansivity and linked prior to the presence of a sub-glass transition [15].

In an attempt to obtain $A_{0}$ beyond the range of actual $p V T$ measurements, the molar volume, thermal expansivity and isothermal compressibility were extrapolated to lower temperatures using the well established equations of state [4,5]. $A_{0}$ (old) calculated, for example, for poly(oxy-1,4-phenylenesulfonyl-1,4-phenylene-oxy-1,4-phenylene-(1-methylidene)-1,4-phenylene) [20] at 40 K increased by a factor of 100 . This increase is expected because of the simplifying assumptions of a constant number of vibrators in this calculation. The value of $A_{0}$ (new) increased, however, also, but only tenfold. This abnormal increase in

Table 4 Calculated $A_{0}$ for a ṣemicrystalline Polytetrafluoroethylene*

| Temp. K | $\begin{aligned} & V \times 10^{5}, \\ & \mathrm{~m}^{3} / \mathrm{mol} \end{aligned}$ | $\begin{gathered} \alpha \times 10^{10} \\ K^{-1} \end{gathered}$ | $\begin{gathered} \beta \times 10^{10} \\ \mathrm{~Pa}^{-1} \end{gathered}$ | $\begin{gathered} C_{p} \\ \mathrm{~J} \cdot \mathrm{~K}^{-1} \mathrm{~mol}^{-1} \end{gathered}$ | $\begin{gathered} C_{v} \\ \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~mol}^{-1} \end{gathered}$ | $\begin{gathered} A_{0} \times 10^{3} \\ \text { (new), } \\ \mathrm{K} \mathrm{~mol} \mathrm{~J}{ }^{-1} \end{gathered}$ | $\begin{gathered} A_{0} \times 10^{3} \\ (\text { old }) \\ \mathrm{K} \mathrm{~mol} \mathrm{~J} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 300.00 | 2.318 | 3.992 | 3.237 | 45.370 | 41.946 | 5.642 | 10.064 |
| 310.00 | 2.328 | 4.033 | 3.449 | 46.020 | 42.618 | 5.357 | 9.406 |
| 320.00 | 2.337 | 4.073 | 3.662 | 46.950 | 43.562 | 5.074 | 8.716 |
| 330.00 | 2.347 | 4.112 | 3.876 | 47.860 | 44.481 | 4.822 | 8.112 |
| 340.00 | 2.356 | 4.151 | 4.090 | 48.760 | 45.385 | 4.595 | 7.577 |
| 350.00 | 2.366 | 4.189 | 4.306 | 49.630 | 46.255 | 4.392 | 7.105 |
| 360.00 | 2.376 | 4.227 | 4.523 | 50.490 | 47.111 | 4.207 | 6.683 |
| 370.00 | 2.386 | 4.264 | 4.740 | 51.350 | 47.963 | 4.038 | 6.300 |
| 380.00 | 2.396 | 4.301 | 4.959 | 52.170 | 48.773 | 3.886 | 5.961 |
| 390.00 | 2.407 | 4.337 | 5.177 | 52.990 | 49.580 | 3.745 | 5.652 |

[^2]$A_{0}$ (new) may indicate that not only Eq. (1) but also the extrapolation of $p V T$ data to lower temperatures (and the used equations of state) may need more extensive experimental checks. In this paper $A_{0}$ (new) for glassy polymers was for this reason only calculated over the range of actual $p V T$ measurements.

In semicrystalline and crystalline polymer, the difficulty of premelting is encountered, making data close to the melting temperature suspect. For the semicrystalline polymers listed in Table 2, the range of $p V T$ measurement over which $A_{0}$ was calculated is thus limited to temperatures much below equilibrium melting. The results for semicrystalline polytetrafluoroethylene are depicted in Table 4. The equilibrium melting temperature of this polymer is about 605 K [13]. The $A_{0}$ obtained from $300-390 \mathrm{~K}$ originate from a comparison of Weir's [10] and Zoller's [12] data that agreed within about $2 \%$. Beyond 390 K the $A_{0}$ values reported earlier were used [1]. For the temperature range of $300-500 \mathrm{~K}$ the average $A_{0}($ new $)$ was $4.18 \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / \mathrm{J}$ compared to $6.2 \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / \mathrm{J}$ for $A_{0}($ old $)$. Due to the wider temperature range the reported $A_{0}$ (old) is about $9 \%$ higher than that reported earlier for the limited temperature range of $380-500 \mathrm{~K}$ [1]. As for the glassy polymers, this semicrystalline polymer shows more than 100 K below $T_{m}$ (but above $T_{g}$ ) a similar, slow, decrease in $A_{0}$ (new). The change of $A_{0}$ (new) with temperature is much less than $A_{0}$ (old). Using an average value over the 200 K wide temperature range leads to an error comparable to the fluctuations from polymer to polymer for the universal constant.
$A_{0}$ values for poly(l-butane) and nylon 6 were much higher than the other semicrystalline polymers. There seems to be no present explanation for these unusually high $A_{0}$ values. One can compare the data for poly(1-butene) to the more "normal" ones for poly(4-methyl-1-pentene). The expansion coefficients of these two polymers were within $6 \%$ of each other [8]. The higher $A_{0}$ must thus be largely

Table 5 Calculated $A_{0}$ for crystalline polyethylene

| Temp., <br> $\mathbf{K}$ | $V_{c} \times 10^{5}$, <br> $\mathrm{m}^{3} / \mathrm{mol}$ | $\alpha_{c} \times 10^{4}$ <br> $\mathrm{~K}^{-1}$ | $\beta_{c} \times 10^{4}$, <br> $\mathbf{P a}^{-1}$ | $C_{p}$, <br> $\mathbf{J ~ K}^{-1} \mathrm{~mol}^{-1}$ | $\mathrm{J} \mathrm{K}^{-1}$, <br> $\mathrm{mol}^{-1}$ | $A_{0} \times 10^{3}$ <br> (new), <br> $\mathrm{K} \mathrm{mol} \mathrm{J}^{-1}$ | $A_{0} \times 10^{3}$ <br> $(\mathrm{old})$, <br> K mol J |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 290.00 | 1.396 | 1.669 | 1.432 | 21.210 | 20.422 | 2.050 | 2.504 |
| 300.00 | 1.399 | 2.206 | 1.509 | 21.800 | 20.447 | 3.225 | 3.934 |
| 310.00 | 1.402 | 2.739 | 1.600 | 22.380 | 20.342 | 4.437 | 5.441 |
| 320.00 | 1.406 | 3.267 | 1.709 | 22.950 | 20.139 | 5.583 | 6.915 |
| 330.00 | 1.411 | 3.791 | 1.835 | 23.560 | 19.913 | 6.590 | 8.254 |
| 340.00 | 1.417 | 4.308 | 1.987 | 24.300 | 19.800 | 7.376 | 9.293 |
| 350.00 | 1.423 | 4.819 | 2.170 | 25.250 | 19.919 | 7.910 | 9.905 |
| 360.00 | 1.431 | 5.322 | 2.393 | 26.510 | 20.414 | 8.176 | 9.990 |
| 370.00 | 1.439 | 5.817 | 2.666 | 28.130 | 21.374 | 8.189 | 9.567 |



Fig. 1 Variation of $A_{0}$ as a function of crystallinity for polyethylene at several temperatures as derived from $p V T$ data
attributed to the lower compressibility of poly(1-butene) when compared to poly(4-methyl-1-pentene).

Table 5 shows the results of calculations for crystalline polyethylene. They show a larger change with temperature and the opposite trend than those of polytetrafluoroethylene (Table 4). The equilibrium melting temperature of polyethylene is much closer to the temperature range of interest ( 416.4 K ). The crystalline specific volume of polyethylene calculated from the two-phase model [Eqs (5-7)], was similar to that derived directly from X-ray data on crystals ( $V_{c}$ from X-ray data varies between $1.406 \times 10^{-5}$ at 290 K to $1.440 \times 10^{-5} \mathrm{~m}^{3} / \mathrm{mol}$ at 370 K ) [21], but the thermal expansivity of Table 5 shows a larger increase with temperature than the corresponding X -ray results, which change in expansivity from $2.752 \times 10^{-4}$ to $3.375 \times 10^{-4} \mathrm{~K}^{-1}$, respectively. The expansivity of the amorphous polyethylene obtained by extrapolations of $V_{a}(0, T)$ from the melt showed only a moderate increase over this temperature range and could not be the reason for the disagreement (it changes from $5.068 \times 10^{-4} \mathrm{~K}^{-1}$ at 290 K to $6.593 \times 10^{-4} \mathrm{~K}^{-1}$ at 370 K ). The high expansivity for crystalline polyethylene obtained by extrapolation of $p V T$ data on semicrystalline polyethylene may perhaps result from some premelting. The two-phase model would then be inadequate if the crystallinity decreases at higher temperatures. In fact, a decrease in crystallinity with increasing temperature was already considered as the cause of the deviations observed between calculated and experimental isotherms reported for semicrystalline linear polyethylene using the two-phase model [4].

Assuming that the two-phase model holds, $A_{0}$ (new) was also calculated using the experimental $C_{p}$ data of linear polyethylene at several crystallinities, obtained from
our ATHAS data bank. Figure 1 shows the variation of the so-calculated $A_{0}$ as a function of crystallinity at several temperatures. At $290 \mathrm{~K} A_{0}$ shows the expected slight decrease with increase in crystallinity. This trend reverses at higher temperatures and the increase in $A_{0}$ with an increase in crystallinity can be clearly seen at higher crystallinities. Compared to the larger variation of $A_{0}$ (new) for crystalline polyethylene, $A_{0}$ (new) for liquid polyethylene extrapolated from measurements above the melting temperature to lower temperatures ( $290-370 \mathrm{~K}$ ) varied much less (between $3.89 \times 10^{-3}$ to $\left.4.14 \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / \mathrm{J}\right)$. These observations further support the fact that the large variation in $A_{0}$ with temperature is associated with the crystalline regions in polyethylene.
Finally the $A_{0}$ (new) was used for the computation of $C_{p}$ from $C_{v}$ obtained from the approximate vibrational spectrum using Eq. (4a). Figure 2 shows the results. An average and RMS deviation of $(1.9 \pm 3.0) \%$ was obtained over the temperature range 4.0 to 410 K by using the new equation [Eq. (4a)] with $A_{0}$ (new) of Table 2. The old equation [Eq. (1)] with the same $A_{0}$ (new) gave superficially a slightly better fir [average and RMS deviation ( $1.06 \pm 2.8$ )\%]. It must be observed, however, that the erroneous upturn of $A_{0}$ (old) at higher temperatures accidentally fits the observed increase in heat capacity due to premelting, discussed above. Both errors are within the experimental error limit considered usually to be $\pm 3 \%$ [3]. The previously calculated values [22] showed the larger error ( $1.6 \pm 4.9$ ) \% for the temperature range 4.0 to 450 K with Eq. (1) and an $A_{0}$ of $4.85 \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / \mathrm{J}$. At low temperatures (below 200 K ) the difference between $C_{v}$ and $C_{p}$ is small and all methods appear to be equally good. The erroneous, sharp upturn above the melting temperature, for the calculated $C_{p}$ does not show-up when using Eq. (4a). In


Fig. 2 Heat capacity $C_{p}$ of crystalline polyethylene from $C_{v}$ calculated using the approximate frequency spectrum and Eq. (4a) with an average $A_{0}$ (new) of $3.18 \times 10^{+3}(\mathrm{~K} \mathrm{~mol}) / \mathrm{J}$

Table 6 Deviation of $C_{p}$ calculated for crystalline polyethylene using different $A_{0}$ (new) values, from $C_{p}$ experimental

| Temp., <br> K | $C_{p}{ }^{a}$ <br> (experimental), <br> $\mathrm{J} / \mathrm{K} \mathrm{mol}$ | $C_{p}{ }^{b}$ <br> (calculated), <br> $\mathrm{J} / \mathrm{K} \mathrm{mol}$ | Deviation, <br> $\%$ | $C_{p}{ }^{c}$ <br> (calculated), <br> $\mathrm{J} / \mathrm{K} \mathrm{mol}$ | Deviation, <br> $\%$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 290 | 21.200 | 21.206 | 0.038 | 21.206 | 0.03 |
| 300 | 21.811 | 21.910 | 0.45 | 22.461 | 2.98 |
| 310 | 22.380 | 22.629 | 1.11 | 23.904 | 6.80 |
| 320 | 22.947 | 23.347 | 1.74 | 25.511 | 11.17 |
| 330 | 23.564 | 24.074 | 2.16 | 27.272 | 15.73 |
| 340 | 24.302 | 24.813 | 2.10 | 29.116 | 19.80 |
| 350 | 25.253 | 25.559 | 1.21 | 30.935 | 22.5 |
| 360 | 26.508 | 26.315 | -0.73 | 32.582 | 22.91 |
| 370 | 28.132 | 27.069 | -3.77 | 33.917 | 20.56 |
| 380 | 30.125 | 27.820 | -7.65 | - | - |
| 390 | 32.357 | 28.560 | -11.73 | - | - |
| 400 | 34.485 | 29.294 | -15.05 | - | - |
| 410 | 35.830 | 30.026 | -16.20 | - | - |

${ }^{a}$ Recommended experimental $C_{p}$, collected in the ATHAS data bank.
${ }^{b} C_{p}$ calculated from $C_{E}$ using $A_{0}$ (new) at 290 K .
${ }^{c} C_{p}$ calculated from $C_{n}$ using $A_{0}$ (new) values at the corresponding temperature.
addition, the computed $C_{p}$ values remain within the limit of the Dulong Petit rule up to 1000 K when using $A_{0}$ (new).

The data of Table 5 and Fig. 1 permit one further analysis of the cause of the abnormal increase in $A_{0}$ (new). Assuming that $A_{0}$ (new) changes little with temperature from 290 K to melting one can calculate a "vibration only" $C_{p}$ based on the detailed analysis of heat capacity at lower temperature. These data are listed in column 3 of Table 6 and compared with the varying $A_{0}$ (new) in column 5. Fit between experimental and calculated $C_{p}$ is now good to 360 K instead to 290 K (column 3). Over the final 50 K the experimental $C_{p}$ is however much larger. We expect the solution to these observations to be as follows: 1. The $p V T$ data extrapolated from semicrystalline polyethylene contain, as discussed above, contributions from premelting, leading thus to erroneously high $C_{p} .2$. The $C_{p}$ (experimental) data were derived directly from close to $100 \%$ crystalline polymer and seemed to show little to no premelting and can thus not be represented by the $A_{0}$ (new) above 290 K . 3. Making the reasonable assumption that $A_{0}$ (new) is constant from $290-410 \mathrm{~K}$, there is now a negative error, the calculated $C_{p}$ is too low. This may be a first indication that even $100 \%$ crystalline polyethylene shows an increase beyond vibrational contributions in its heat capacities before melting. One expects this increase to be caused by introduction of defects and large amplitude

Table 7 Calculated $A_{0}$ for crystalline Polypropylene

| Temp., <br> K | $V_{c} \times 10^{5}$, <br> $\mathrm{m}^{3} / \mathrm{mol}$ | $\alpha_{c} \times 10^{4}$, <br> $\mathrm{K}^{-1}$ | $\beta_{c} \times 10^{10}$, <br> $\mathrm{Pa}^{-1}$, | $\mathrm{J} \mathrm{K}_{p}$, <br> $\mathrm{mol}^{-1}$ | $\mathrm{J} \mathrm{K}_{\boldsymbol{v}}$, <br> $\mathrm{mol}^{-1}$ | $A_{0} \times 10^{3}$ <br> $(\mathrm{new})$, <br> K mol J | $A_{0} \times 10^{3}$ <br> $(\mathrm{old})$, <br> $\mathrm{K} \mathrm{mol} \mathrm{J}^{-1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 300.00 | 4.474 | 2.592 | 1.114 | 68.240 | 60.147 | 6.188 | 7.699 |
| 310.00 | 4.486 | 2.745 | 1.153 | 70.950 | 61.861 | 6.399 | 7.741 |
| 320.00 | 4.499 | 2.896 | 1.227 | 73.770 | 63.925 | 6.418 | 7.513 |
| 330.00 | 4.513 | 3.045 | 1.335 | 76.680 | 66.333 | 6.282 | 7.087 |
| 340.00 | 4.528 | 3.192 | 1.476 | 79.670 | 69.044 | 6.038 | 6.544 |
| 350.00 | 4.544 | 3.337 | 1.650 | 82.730 | 72.000 | 5.728 | 5.953 |
| 360.00 | 4.560 | 3.479 | 1.857 | 85.860 | 75.157 | 5.383 | 5.360 |
| 370.00 | 4.578 | 3.619 | 2.094 | 89.040 | 78.445 | 5.032 | 4.800 |

conformational motion of the molecules. 4. Recalculation of the error with $A_{0}$ (new) from 4.0 up to 360 K taken to be constant at its 290 K value of $2.05 \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / \mathrm{J}$ leads to errors of $(1.5 \pm 1.411) \%$.

For crystalline polypropylene $A_{0}$ (new) values were obtained over a range of temperature ( $300-370 \mathrm{~K}$ ) that is again further away from the 460.7 K equilibrium melting temperature (Table 7). Variation in $A_{0}$ (both new and old) with temperature was less than that observed in crystalline polyethylene. This indicates that the contribution of premelting towards increasing the $A_{0}$ is significantly less in polypropylene. The data show furthermore that the $A_{0}$ (new) varies less than $A_{0}$ (old) i.e., Eq. (4a) is an improvement over Eq. (1). For semicrystalline polypropylene of $69.6 \%$ crystallinity an average $A_{0}$ (new) value of $(5.2 \pm 0.4) \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / \mathrm{J}$ was obtained over this temperature range with an $A_{0}$ of liquid polypropylene of $(5.7 \pm 0.5) \times 10^{-3}(\mathrm{k} \mathrm{mol}) / \mathrm{J}$.

## References

1 J. Grebowicz and B. Wunderlich, J. Thermal Anal. 30 (1985) 229.
2 W. Nernst and F. A. Lindemann, Z. Electrochem., 17 (1911) 817.
3 B. Wunderlich and S. Z. D. Cheng, Gazzetta Chimica Italiana, 116 (1986) 345.
4 R. K. Jain and R. Simha, J. Polym. Sci., Polym. Phys. Ed., 17 (1979) 1929.
5 O. P. Pahuja and V. S. Nanda, J. Macromol. Sci. Phys., B25(4) (1986) 419.
6 Y. Tsujita, T. Nose and T. Hata, Polym. J, 5 (1973) 201.

7 O. Olabisi and R. Simha, Macomolecules, 8 (1975) 206.

8 P. Zoller, J. Appl. Polym. Sci., 23 (1979) 1057.
9 P. Zoller, J. Appl. Polym. Sci., 23 (1979) 1051.
10 C. E. Weir, J. Res. Natl. Bur. Stand., 53 (1954) 245.

11 P. Zoller, J. Appl. Polym. Sci., 21 (1977) 3129.
12 P. Zoller, J. Appl. Polym. Sci., 22 (1978) 633.
13 S. F. Lau, H. Suzuki and B. Wunderlich, J. Polym. Sci., Polym. Phys. Ed., 22 (1984) 379.
14 J. E. McKinney and R. Simha, Macromolecules, 7 (1974) 894.

15 A. Quach, P. S. Wilson and R. Simha, J. Macromol. Sci., Phys., B9 (3) (1974) 533.
16 P. Zoller, J. Polym. Sci., Polym. Phys. Ed., 20 (1982) 1453.

17 A. Quach and R. Simha, J. Appl. Phys., 42 (1971) 4592.

18 P. Heydemann and H. D. Guicking, KolloidZ. Z. Polym., 193 (1963) 16.

19 P. Zoller, J. Macromol. Sci. Phys., B18 (3) (1980) 555.

20 P. Zoller, J. Polym. Sci., Polym. Phys. Ed., 16
(1978) 1261.
21 B. Wunderlich, "Macromolecular Physics, Vol. 3, Crystal Melting", Academic Press, New York, 1980, p. 47.
22 J. Grebowicz, H. Suzuki and B. Wunderlich, Polymer, 26 (1985) 561.

Zusammenfassung - Es wurde eine Abänderung der Nernst-Lindemann Gleichung vorgeschlagen, mit deren Hilfe für feste, lineare Makromoleküle errechnete Wärmekapazitäten bei konstantem Druck ( $C_{p}$ ) in Wärmekapazitäten bei konstantem Volumen ( $C_{p}$ ) umgerechnet werden können. Zur Ableitung der molaren Konstanten $A_{0}$ in dem sich wiederholenden Teil der Gleichung wurde die variable Anzahl der erregten Schwinger bei verschiedenen Temperaturen berücksichtigt. Mit der neuen Gleichung wird es möglich, die $C_{p}$ fester Polymere für einen breiten Temperaturbereich zu errechnen. Die Konstante wurde für die festen Polymere auf Grund des ermittelten thermischen Ausdehnungsvermögens und der isothermen Kompressibilität sowie der der Literatur entnommenen Wärmekapazitätsangaben berechnet. Aus Angaben von 22 festen Polymeren wurde für $A_{0}(n e u)$ ein Durchschnittswert von $(3,9 \pm 2,4) \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / J$ erhalten. Verfügt man zur Berechnung von $A_{0}$ über keine experimentellen Werte für Kompressibilität und Ausdehnungsvermögen, so kann dieser durchsenittswert als universale Konstante angewendet werden. Die verbleibende Temperaturabhängigkeit von $A_{0}$ (neu) wird besprochen und Beispielrechnungen für Polyäthylen gegeben. Für Polyäthylen wurden im Temperaturbereich 290 bis 410 K Effekte durch Vorschmelzen und Bewegungen mit großer Amplitude festgestellt.

Реаюме - Предложена модификация уравнения Нернста-Линдеманна, используемого для превращения вычисленных теплоемкостей при постоянном давлении ( $C_{p}$ ) в теплоемкости при постоянном объеме ( $C_{v}$ ) для твердых, линейных макромолекул. В этом уравнении константа $A_{0}$ на моль повторяющегося звена выведена с учетом переменного числа вибраторов, возбужденных при различных гемпературах. С помощью нового уравнения представляется возможным вычислять $C_{p}$ для твердых полимеров в более широком температурном интервале. Константа $A_{0}$ для твердых полимеров вычислена, исходя из литературных значений коэффициентов термического расширения, изотермической сжимаемости и теплоемкости. Исходя из данных для 22 твердых полимеров, было получено среднее значение константы $A_{0}$ (новая) равное $(3,9 \pm 2,4) \cdot 10^{-3} \mathrm{~K} \cdot$ моль/дж. Такое среднее значение может быть использовано в качестве универсальной константы при вычислении $A_{0}$ в случае отсутствия экспериментальных данных по сжимаемости и расширению. Обсуждено изменение новой $A_{0}$ от температуры, а расчеты показаны на примере полиэтилена. Для полиэтилена в интервале температур $290-410 \mathrm{~K}$ обнаружены эффекты предплавления и возможное движение с большой амплитудой.


[^0]:    * Number in parenthesis indicate the number of heavy atoms in repeating unit.
    ** Melting temperature used in $A_{0}$ calculation is our data's bank recommended data, see Ref. [3].

[^1]:    * Number in parenthesis indicate the number of heavy atoms in the repeating unit. (c) indicates fully crystalline samples and (s) semicrystalline sample.
    ** Melting temperature used in $A_{0}$ calculation is our data bank's recommended data, see Ref. [3].
    *** As can be seen from the discussion section, the $A_{0}$ values above 290 K have a contribution from premelting. A better $A_{0}$ (new) value would be $2.05 \times 10^{-3}(\mathrm{~K} \mathrm{~mol}) / \mathrm{J}$ at 290 K .

[^2]:    * Estimated crystallinity $51 \%$.

